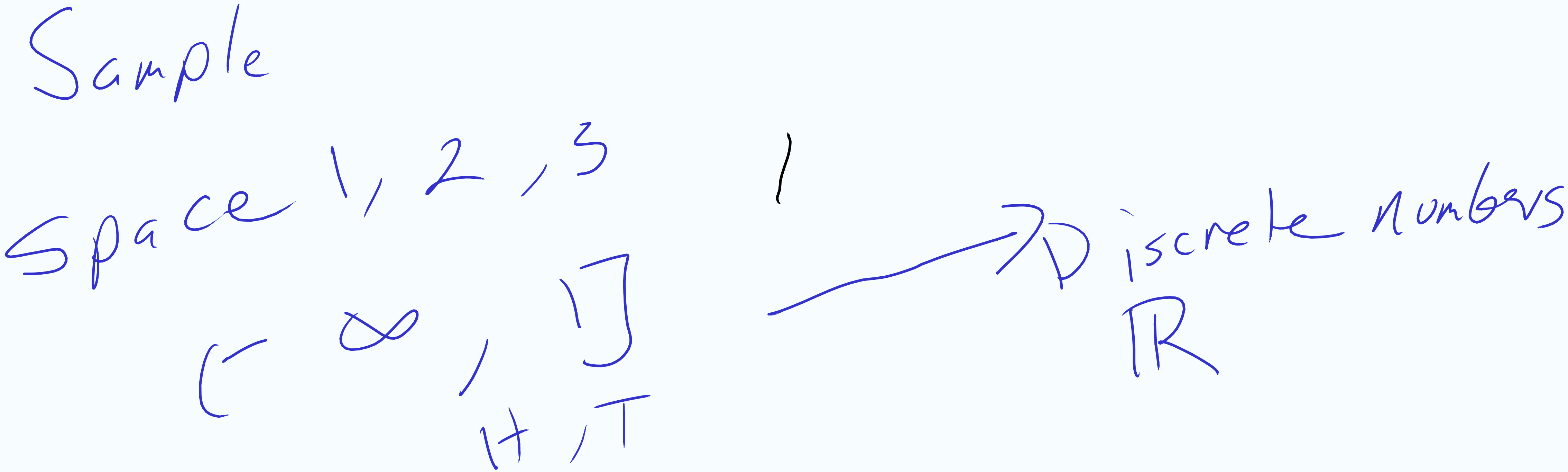
REVIEW:

RANDOM VARIABLES AND DISTRIBUTION FUNCTIONS

- Review Random Variables
- Discrete Random Variables
- Binomial Distribution
- Poisson Distribution
- Continuous Random Variables
- Uniform Distribution
- Normal Distribution

RANDOM VARIABLES

A random variable is function that maps the sample space to real value.



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DISCRETE RANDOM VARIABLES

A random variable is considered to be discrete if it can only map to a finite or countably infinite number of distinct values.

PMF

$0 \le P(Y=y) \le 1$

The probability mass function of discrete variable can be represented by a formula, table, or a graph. The Probability of a random variable Y can be expressed as $P(Y = y)$ for all values of y.

$y =$	0	1	2	3	4
	1/10	1/10	1/5	1/5	2/5

CDF

Distribution Function.

The cumulative distribution function provides the $P(Y \leq y)$ for a random variable Y.

	0	1	2	3	4
PMF	1/10	1/10	1/5	1/5	2/5
CDF	1/10	1/5	2/5	3/5	5/5

EXPECTED VALUE

The *expected value* is the value we expect when we randomly sample from population that follows a specific distribution. The expected value of Y is

$$E(Y) = \sum_{y} yP(y)$$

$H = 1 \quad T = 0$

$0 \cdot 0.5 + 1 \cdot 0.5$

$E(Y) = 0.5$

VARIANCE

The *variance* is the expected squared difference between the random variable and expected value.

$$Var(Y) = \sum_y \{y - E(Y)\}^2 P(y)$$

$$Var(Y) = E(X^2) - E(X)^2$$

$$\sum_Y (Y - E(Y)) P(Y) \qquad E(Y) = \mu$$

$$\sum_Y (Y - \mu) P(Y)$$

$$\sum_Y (Y P(Y) - \mu P(Y)) \qquad E(Y) = \sum Y P(Y)$$

$$\sum_Y Y P(Y) - \sum_Y \mu P(Y)$$

$$\sum_Y Y P(Y) - \mu \sum_Y P(Y)$$

$$E(Y) - \mu$$

$$\mu - \mu = 0$$

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KNOWN DISTRIBUTIONS

Distribution	Parameter(s)	PMF $P(Y = y)$
Bernoulli	p	p
Binomial	n and p	$\binom{n}{y} p^y (1-p)^{n-p}$
Geometric	p	$(1-p)^{y-1} p$
Negative Binomial	r and p	$\binom{y-1}{r-1} p^{r-1} (1-p)^{y-r}$
Hypergeometric	N, n, and r	$\frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$
Poisson	λ	$\frac{\lambda^y}{y!} e^{-\lambda}$

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BINOMIAL DISTRIBUTION

An experiment is said to follow a binomial distribution if

1. Fixed n
2. Each trial has 2 outcomes
3. The probability of success is a constant p
4. The trials are independent of each

$$\binom{n}{x} = \frac{n!}{(n-x)! \cdot x!}$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

EXPECTED VALUE OF A BINOMIAL DISTRIBUTION

$E(X) = np$

$$\sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$\sum_{x=0}^{n} x \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x}$$

$$\sum_{x=1}^{n} x \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x}$$

$$\frac{m!}{(m-y)!\,y!} p^y (1-p)^{m-y}$$

$$\sum_{x=1}^{n} \frac{n!}{(n-x)!(x-1)!} p^x (1-p)^{n-x}$$

$$z = x - 1$$

$$\sum_{z=0}^{n-1} \frac{n!}{(n-(z+1))!\, z!} p^{z+1} (1-p)^{n-(z+1)}$$

$$\sum_{z=0}^{n-1} \frac{n!}{(n-(z+1))!\, z!} p \cdot p^{z} (1-p)^{n-(z+1)}$$

$$p \sum_{z=0}^{n-1} \frac{n!}{(n-(z+1))!\, z!} p^{z} (1-p)^{n-z-1}$$

$$p \sum_{z=0}^{n-1} \frac{n!}{((n-1)-z)!\, z!} p^{z} (1-p)^{(n-1)-z}$$

$$p \sum_{z=0}^{n-1} \frac{n \cdot (n-1)!}{((n-1)-z)!\, z!} p^{z} (1-p)^{n-1-z}$$

$$np \sum_{z=0}^{n-1} \underbrace{\frac{(n-1)!}{((n-1)-z)!\, z!} p^z (1-p)^{(n-1)-z}}$$

$$Z \sim Bn(n-1, p)$$

$$np$$

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POISSON DISTRIBUTION

The poisson distribution describes an experiment that measures that occurrence of an event at specific point and/or time period.

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

EXPECTED VALUE OF A POISSON DISTRIBUTION

$0 < x < \infty$

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$E(X) = \lambda$$

$$E(X) = \sum_{x=0}^{\infty} x \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$\sum_{x=1}^{\infty} \frac{x e^{-\lambda} \lambda^{x}}{x!}$$

$$\sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^{x}}{(x-1)!}$$

$$\frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$\sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda \lambda^{x-1}}{(x-1)!} \qquad z = x-1$$

$$\lambda \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^{x-1}}{(x-1)!}$$

$$\lambda \sum_{z=0}^{\infty} \frac{e^{-\lambda} \lambda^{z}}{z!} = \lambda$$

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CONTINUOUS RANDOM VARIABLES

A random variable X is considered continuous if the $P(X = x)$ does not exist.

CDF

The cumulative distribution function of X provides the $P(X \leq x)$, denoted by $F(x)$, for the domain of X.

Properties of the CDF of X:

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(x)$ is a nondecreaseing function

PDF

The probability density function of the random variable X is given by

$$f(x) = \frac{dF(x)}{d(x)} = F'(x)$$

wherever the derivative exists.

Properties of pdfs:

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $P(a \leq X \leq b) = P(a < X < b) = \int_a^b f(x)dx$

EXPECTED VALUE

The expected value for a continuous distribution is defined as

$$E(X) = \int xf(x)dx$$

The expectation of a function $g(X)$ is defined as

$$E\{g(X)\} = \int g(x)f(x)dx$$

EXPECTED VALUE PROPERTIES

1. $E(c) = c$, where c is constant
2. $E\{cg(X)\} = cE\{g(X)\}$
3. $E\{g_1(X) + g_2(X) + \cdots + g_n(X)\} = E\{g_1(X)\} + E\{g_2(X)\} + \cdots$

VARIANCE

The variance of continuous variable is defined as

$$Var(X) = E[\{X - E(X)\}^2] = \int \{X - E(X)\}^2 f(x)dx$$

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UNIFORM DISTRIBUTION

A random variable is said to follow uniform distribution if the density function is constant between two parameters.

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

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NORMAL DISTRIBUTION

A random variable is said to follow a normal distribution if the the frequency of occurrence follow a Gaussian function.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\}$$

EXPECTED VALUE