Statistical Estimators

Estimators

An *estimator* is an operation computing the value of an estimate, that targets the parameter, using measurements from a sample.

Estimator Definition

Given a *random sample*, $X = (X_1, \ldots, X_n)^{\mathrm{T}}$ from a distribution $F(\theta)$, an estimator is defined as

$$\hat{\theta} = h(X)$$

Example Estimator

Given a *random sample*, $X = (X_1, \dots, X_n)^{\mathrm{T}}$ from a distribution $F(\theta)$

Methods of Finding Estimators

- Maximum Likelihood Estimator
- Method of Moments Estimator
- Bayesian Estimtors

Evaluating Estimators

- Mean Squared Error
- Unbiased Estimator
- Sufficiency

Unbiased Estimator

An unbiased estimator $\widehat{\theta}$ is an estimator that satisfies the following condition:

$$E(\widehat{\theta}) = \theta$$

Bias

The bias of an estimator is defined as

$$B(\widehat{\theta}) = E(\widehat{\theta}) - \theta$$

Mean Square Error

The mean square error of an estimator is $\hat{\theta}$ is given as

$$\begin{aligned} MSE(\hat{\theta}) &= E\{(\hat{\theta} - \theta)^2\} \\ &= Var(\hat{\theta}) + B(\hat{\theta})^2 \end{aligned}$$

Is $\bar{X}$ an unbiased estimator of μ?

Let $X_1, \ldots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$, find the bias of $\bar{X}$.

Why is S^2 divided by $n-1$ instead of n?

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, find the bias of S^2.

Problem

Let X_1, X_2, X_3 follow and exponential distribution with mean and variance λ and λ^2, respectively. Using the following estimators:

- $\widehat{\theta}_1 = X_1$
- $\widehat{\theta}_2 = \frac{X_1+X_2}{2}$
- $\widehat{\theta}_3 = \frac{X_1+2X_2}{3}$
- $\widehat{\theta}_4 = \frac{X_1+X_2+X_3}{3}$

Identify which estimator:

1. Is unbiased?

Likelihood Function

Using the joint pdf or pmf of the sample X, the likelihood function is a function of θ, given the observed data $X = x$, defined as

$$L(\theta|x) = f(x|\theta)$$

If the data is iid, then

$$f(x|\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

Log-Likelihood Function

If $\ln\{L(\theta)\}$ is monotone of θ, then maximizing $\ell(\theta) = \ln\{L(\theta)\}$ will yield the maximum likelihood estimators.

Binomial Distribution

Let $X_1, \dots, X_n \overset{iid}{\sim} \text{Bin}(n, p)$

Poisson Distribution

Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Pois}(\lambda)$

Normal Distribution

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

Exponential Distribution

Let $X_1, \ldots, X_n \overset{iid}{\sim} Exp(\lambda)$