Maximum Likelihood Estimators

Likelihood Function

Using the joint pdf or pmf of the sample X, the likelihood function is a function of θ, given the observed data $X = x$, defined as

$$L(\theta|x) = f(x|\theta)$$

If the data is iid, then

$$f(x|\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

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Log-Likelihood Function

If $\ln\{L(\theta)\}$ is monotone of θ, then maximizing $\ell(\theta) = \ln\{L(\theta)\}$ will yield the maximum likelihood estimators.

Maximum log-Likelihood Estimator

The maximum likelihood estimator are the estimates of θ that maximize $\ell(\theta)$.

Binomial Distribution

Let $X_1, \dots, X_n \overset{iid}{\sim} \mathrm{Bin}(n, p)$

Poisson Distribution

Let $X_1, \dots, X_n \overset{iid}{\sim} \text{Pois}(\lambda)$

Normal Distribution

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

Exponential Distribution

Let $X_1, \dots, X_n \overset{iid}{\sim} Exp(\lambda)$