Bayesian Estimators

Data

Let $X_1, \ldots, X_n \overset{iid}{\sim} F(\theta)$ where $F(\cdot)$ is a known distribution function and θ is a vector of parameters. Let $X = (X_1, \ldots, X_n)^{\mathrm{T}}$, be the sample collected.

Frequentist-based Estimation

Bothe the **maximum likelihood estimator** and **method of moments estimator** are known as frequentist-based estimators since the parameters we are esimating are considered **fixed**.

Bayesian Estimation

The idea of Bayesian estimation is that our parameters θ are random variables instead. Therefore; they follow the probability theory that we have learned so far.

Using the prior distribution, we would like to find the distribution of θ when we incorporate the data X into our distribution. We are looking to find the **posterior distribution**.

Prior Distribution

The prior distribution is the distribution for the random variable θ:

$$\pi(\theta)$$

Likelihood Function

$$L(\theta|x) = f(x|\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

Likelihood X Prior

$$L(\theta|x) \times \pi(\theta)$$

Posterior Distribution

The posterior distribution is the distribution of θ when we condition it on the data X

$$\pi(\theta|x) = \frac{L(\theta|X)\pi(\theta)}{\int L(\theta|X)\pi(\theta)d\theta}$$

Estimators

- Bayesian Estimator
- *maximum a posteriori* (MAP)

Bayesian Estimator

$$\widehat{\theta} = E_{\theta|X}(\theta) = \int \theta \pi(\theta|x) d\theta$$

MAP

$$\tilde{\theta} = \text{argmax}_{\theta} \pi(\theta|x)$$

Conjugate Prior

A conjugate prior distribution is any distribution that result in the posterior distribution having a similar form as the conjugate distribution.

Hyper Parameters

Hyperparameters are the parameters involved in the prior distribution.

Bernoulli Distribution

Let $X_1, \ldots, X_n \overset{iid}{\sim} \mathrm{Bin}(1, p)$

Poisson Distribution

Let $X_1, \ldots, X_n \overset{iid}{\sim} \text{Pois}(\lambda)$

Nomal Distribution

Let $X_1, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$