$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_x}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}$$

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_x}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\} dy$$

$$K = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \qquad \gamma = \frac{1}{2(1-\rho^2)}$$

$$Z_x = \left(\frac{x-\mu_x}{\sigma_x}\right)^2 \qquad Z_y = \left(\frac{y-\mu_y}{\sigma_y}\right)^2$$

$$= \int K \exp\{-\gamma[Z_x^2 - 2\rho Z_x Z_y + Z_y^2]\}\,dy$$

$$K \int \exp\{-\gamma Z_x^2\} \exp\{-\gamma(Z_y^2 - 2\rho Z_x Z_y)\}\,dy$$

$$K \exp\{-\gamma Z_x^2\} \int \exp\{-\gamma(Z_y^2 - 2\rho Z_x Z_y)\}\,dy$$

$$K' = K \exp\{-\gamma Z_x\}$$

$$K' \int \exp\{-\gamma(Z_y^2 - 2 Z_y Z_x \rho)\}\,dy$$

$$k' \int \exp\{-\gamma(z_y^2 - 2 z_y z_x \rho + \rho^2 z_x^2 - \rho^2 z_x^2)\} \, dy$$

$$k' \int \exp\{-\gamma [(z_y - \rho z_x)^2 - \rho^2 z_x^2]\} \, dy$$

$$k' \int \exp\{-\gamma (z_y - \rho z_x)^2\} \exp\{\gamma \rho^2 z_x^2\} \, dy$$

$$k' \exp\{\gamma \rho^2 z_x^2\} \int \exp\{-\gamma (z_y - \rho z_x)^2\} \, dy$$

$$k'' = k' \exp\{\gamma \rho^2 z_x^2\}$$

$$k'' \int \exp\{-\gamma (z_y - \rho z_x)^2\} \, dy$$

$$\gamma = 2(1-\rho^2) \qquad z_y = \left(\frac{y - \mu_x}{\sigma_y}\right)$$

$$dz_y = \frac{dx}{\sigma_y} \qquad k''' = k'' \sigma_y$$

$$k'' \int \exp\left\{-\frac{1}{2} \frac{(z_y - \rho z_x)^2}{1-\rho^2}\right\} \frac{dy}{\sigma_y}$$

$$k''' \int \exp\left\{-\frac{1}{2} \frac{(z_y - \rho z_x)^2}{(1-\rho^2)}\right\} dz_y$$

$$k^{IV} = k''' \sqrt{2\pi(1-\rho^2)}$$

$$k^{IV} \int \frac{1}{\sqrt{2\pi(1-\rho^2)}} \exp\left\{-\frac{1}{2} \frac{(z_y - \rho z_x)^2}{(1-\rho^2)}\right\} dz_y$$

$$z_y \sim N(\rho z_x, (1-\rho^2))$$

$$k^{IV} [1]$$

$$k^{IV} = \frac{1}{\sqrt{2\pi \sigma_x^2}} \exp\{-\gamma z_x^2 + \gamma \rho^2 z_x^2\}$$

$$\frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{ -\gamma z_x^2 (1-\rho^2) \right\}$$

$$\frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{ z_x^2 \frac{\cancel{(1-\rho^2)}}{2\cancel{(1-\rho^2)}} \right\}$$

$$\frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{ -\frac{1}{2} \left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right\}$$

$$X \sim N(\mu_x, \sigma_x^2)$$