Point Estimates

Guesses parameters.

$\bar{X} \longrightarrow \mu$

$T(X) \longrightarrow \theta$

$$PE \pm \underbrace{\text{Margin of Error}}$$

Statistical Intervals

A set of values that may contain/capture the true parameter
(Bayesian) (Frequentist)

$$P(L \leq \theta \leq U) = 1 - \alpha$$

L? U?

α: significance level
This is the probability that your interval does not contain/capture your true parameter.

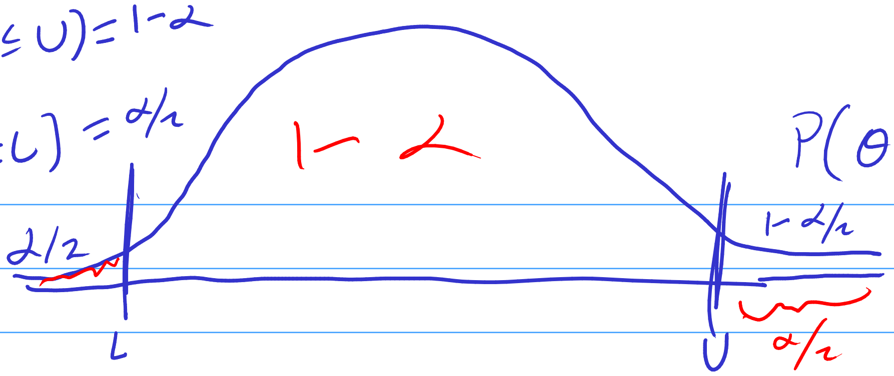
$\alpha = 0.05$

L, U are constructed from a mathematical model such as.
$N(\mu, \sigma^2)$, ...

$P(L \leq \theta \leq U) = 1-\alpha$

$P(\theta \leq L) = \alpha/2$

$P(\theta \leq U) = 1-\alpha/2$



Mathematical Model

$$N(\mu, \sigma^2/n) \qquad \text{What is } \mu\text{?}$$

$$X = \{X_1, \ldots\ldots, X_n\} : \underline{E(X) = \mu \quad Var(X) = \sigma^2}$$

$n \to \infty$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

Via Central Limit Theorem

$$P(L \leq \mu \leq U) = 1-\alpha$$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

$$Z_{\bar{x}} = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$P\left(L \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq U\right) = 1-\alpha$$

$$P\left(Z_{\bar{x},\alpha/2} \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq Z_{x,1-\alpha/2}\right)$$

$P(Z \le Z_{\bar{x}\, \alpha/2}) = \alpha/2$ $\quad$ $P(Z \le Z_{\bar{x}\, 1-\alpha/2}) = 1-\alpha/2$

$\alpha = 0.05$

$Z_{\alpha/2} = -1.96$ $\quad$ $Z_{1-\alpha/2} = 1.96$

$$P\left(-1.96 \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le 1.96\right)$$

$$P\left(-1.96\,\sigma/\sqrt{n} \le \bar{X} - \mu \le 1.96\,\sigma/\sqrt{n}\right) = .95$$

$$-\bar{X} \qquad -\bar{X} \qquad -\bar{X}$$

$$P\left(-\bar{X} - 1.96\,\sigma/\sqrt{n} \le -\mu \le -\bar{X} + 1.96\,\sigma/\sqrt{n}\right) = .95$$

$$P\left(\bar{X} + 1.96\,\sigma/\sqrt{n} \ge \mu \ge \bar{X} - 1.96\,\sigma/\sqrt{n}\right) = .95$$

$$P\left(\bar{X} - 1.96\,\sigma/\sqrt{n} \le \mu \le \bar{X} + 1.96\,\sigma/\sqrt{n}\right) = .95$$

95% CI of μ

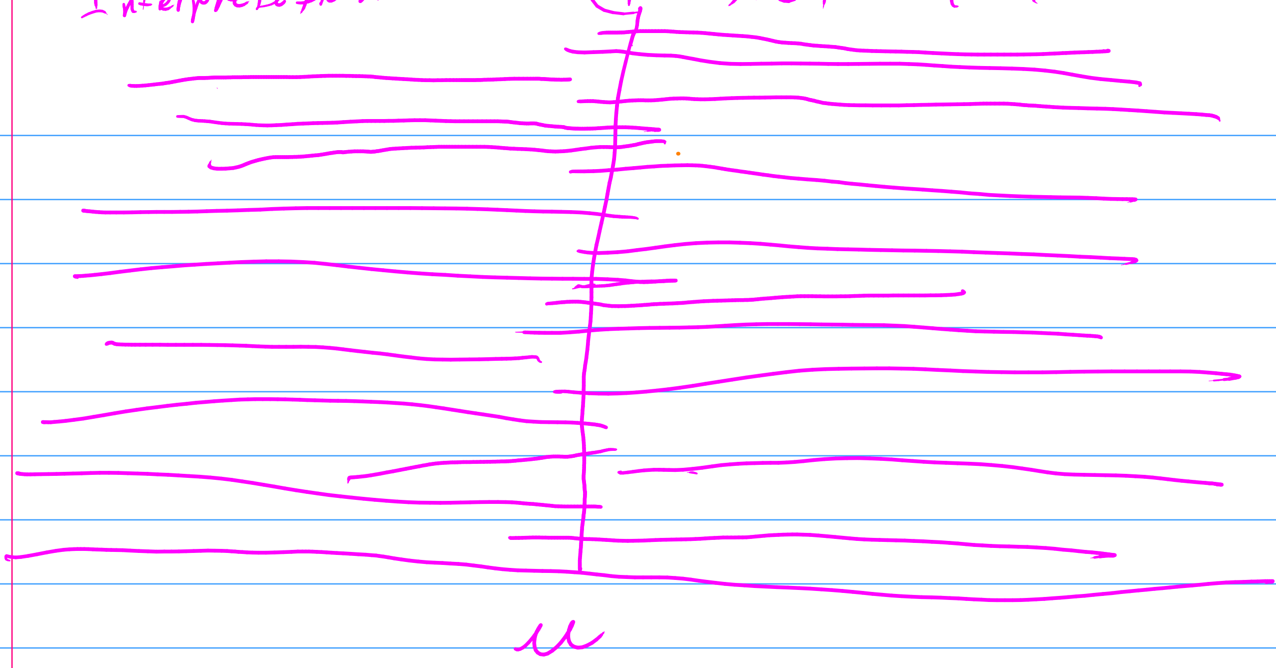
$$\bar{X} \pm 1.96\,\sigma/\sqrt{n}$$

$n \to \infty$

$s^2 \to \sigma^2$

Interpretation

Interpretation $(1-\alpha)\%$ CI 95%



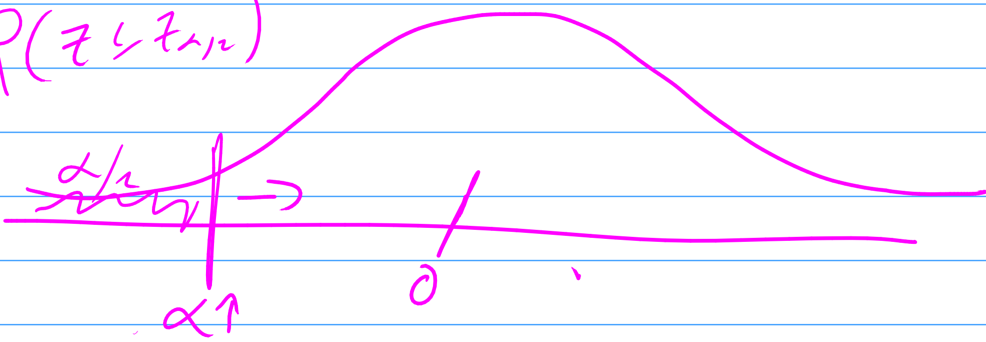
Width of the interval

α value ↑ width ↓ $U - L$

n ↑ width ↓

$$\bar{x} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$$

$P(Z < z_{\alpha/2})$



Confidence $= 1 - \alpha$

$(1-\alpha)100\%$

$MOE = |z_{\alpha/2}|\, \sigma/\sqrt{n}$

Margin of Error