$(1-\alpha)\,100\%$ Confidence Interval

$$P\left(\bar{X} - z_{\alpha/2}\,\sigma/\sqrt{n} \le \mu \le \bar{X} + z_{\alpha/2}\,\sigma/\sqrt{n}\right) = 1-\alpha$$

$$\underset{PE}{\bar{X}} \pm \underset{CV}{z_{\alpha/2}}\ \underset{SE}{\sigma/\sqrt{n}}$$

$(L(-),\ U(+))$

$$P.E. \pm CV \cdot SE$$

Point Estimate — Critical Value — Standard Error

$Y \sim N$

$\bar{X} \pm$ $P(Y \le cv) = 1-\alpha/2$ $SE = \sqrt{Var(\bar{X})}$

Standard Normal $\frac{\sigma}{\sqrt{n}}$

$z_{1-\alpha/2}$

90% CI

$(1-\alpha)\,100\%$

$\alpha = 0.1$

$\alpha/2 = 0.05$

$z_{0.95} = 1.645$

90% CI:

$$\left(\bar{x} - 1.645\,\sigma/\sqrt{n},\ \bar{x} + 1.645\,\sigma/\sqrt{n}\right)$$

95% $\quad z_{.975}$

$\bar{x} \pm 1.96\ \sigma/\sqrt{n}$

99% CI $\quad z_{.995}$

$\bar{X} \pm 2.575\ \sigma/\sqrt{n}$

$z_{1-\alpha/2}$

$\bar{X} \pm z_{\alpha/2}\ \sigma/\sqrt{n}$

$z_{0.005}$

$|-2.575|$

$X \sim N(8, 9)$

$P(X \leq 6)$

z-score

$\frac{X-\mu}{\sigma} = z \sim N(0,1)$

z-table: $P(z \leq z_0)$

$P\left(z \leq \frac{6-8}{3}\right) = P\left(z \leq \frac{-2}{3}\right)$

$P(z \leq -0.67) = 0.2514$

$P(X \leq 6) = 0.2514$

$P(X > 5) = P(X \geq 5)$

$P\left(Z \geq \frac{5-8}{3} = -1\right)$

$P(Z > -1) = 1 - P(Z < -1)$

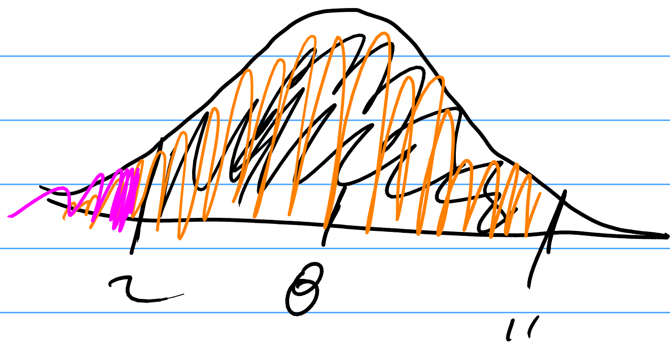
$= 1 - .1587$

$P(X > 5) = .8413$

$X \sim N(8, 9)$

$P(2 \leq X \leq 11)$

$P(X \leq 11) - P(X \leq 2)$



$P(Z \leq 1) - P(Z \leq -2)$

$.8413 - .0228$

$\approx .8185$

$P(Z \leq 5.8)$

$P(Z \geq 2) = P(Z \leq -2)$

$\bar{X} \; \dot{\sim} \; N(\mu, \sigma^2/n)$

by CLT

$n \to \infty$

$$Z_{\bar{x}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$t_\infty = N(0,1)$

$t_{30} \to N(0,1)$

$n = 50 \geq 30$

$s^2 \to \sigma^2$

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim N(\mu, s^2/n)$$

$X_1, \dots, X_n \overset{iid}{\sim}$ Bernoulli (p)

$\hat{p} = \frac{Y}{n}$ $\quad Y = \#$ 1's in $\cancel{X} = X_1, \dots, X_n$

$n \to \infty$

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

$E(X_i) = p \qquad Var(X_i) = p(1-p)$

$\hat{p} = \bar{X}$ $\qquad n \to \infty$

$\hat{p} \; \dot{\sim} \; N(p, p(1-p)/n)$ via CLT

$np > 20$ $\leftarrow$ very important!

$(1-\alpha)100\%$ CI

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

(0.85, 0.95)

We are 95% confident that the true proportion is captured b/w (0.85, 0.95)

95% Credible Interval (Bayesian Approach)

(0.85, 0.95)

With a 95% probability; the true proportion can be as low as 0.85 and as high as 0.95.