Intervals for variance

$$S^2 = \frac{1}{n-1} \sum_{i=1} (x_i - \bar{x})^2$$

$X\%$ Confidence

Identify the bounds that capture the true parameter value

σ^2 = variance

$$\frac{(n-1)\,S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$P\left(L < \frac{(n-1)\,s^2}{\sigma^2} < U \right) = 1 - \alpha$$

$$P\left(\chi^2_{n-1,\,1-\alpha/2} < \frac{(n-1)\,s^2}{\sigma^2} < \chi^2_{n-1,\,\alpha/2} \right) = 1 - \alpha$$

$$P\left(\frac{\chi^2_{n-1,\,1-\alpha/2}}{(n-1)s^2} < \frac{1}{\sigma^2} < \frac{\chi^2_{n-1,\,\alpha/2}}{(n-1)\,s^2} \right) = 1 - \alpha$$

$$\frac{(n-1)\,s^2}{\chi^2_{n-1,\,\alpha/2}} < \sigma^2 < \frac{(n-1)\,s^2}{\chi^2_{n-1,\,1-\alpha/2}}$$

$(1-\alpha)100\%$ CI σ^2

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} , \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} \right)$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\{X_i\}_{i=1}^{n} \overset{iid}{\sim} N(\mu, \sigma^2)$$

$\bar{X}$ $n \geq 30$

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$(1-\alpha)100\%$ CI

$$\bar{X} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$$

$n > 30$

$s^2 \to \sigma^2$

$$\bar{X} \pm z_{\alpha/2}\, s/\sqrt{n}$$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\frac{\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}}{\sqrt{\dfrac{\cancel{(n-1)}\,s^2}{\sigma^2}\Big/\cancel{(n-1)}}} \rightarrow \frac{\dfrac{\bar{x}-\mu}{\cancel{\sigma}/\sqrt{n}}}{s/\cancel{\sigma}} \qquad \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$$

$$P\left(L \le \frac{\bar{X}-\mu}{s/\sqrt{n}} \le U \right) = 1-\alpha$$

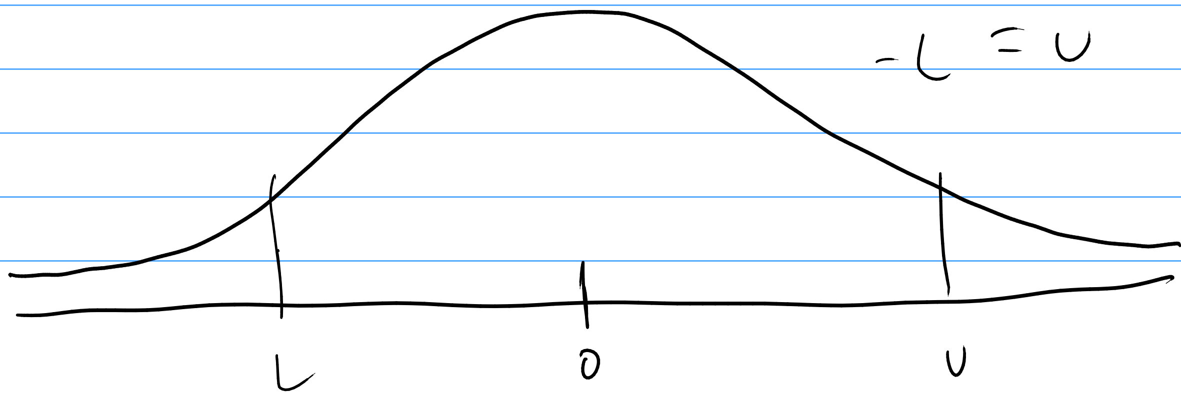
$$P\left(Ls/\sqrt{n} \le \bar{X}-\mu \le Us/\sqrt{n} \right) = 1-\alpha$$

$$P\left(-\bar{X}+Ls/\sqrt{n} \le -\mu \le -\bar{X}+Us/\sqrt{n} \right) = 1-\alpha$$

$$P\left(\bar{X}-Ls/\sqrt{n} > \mu > \bar{X}-Us/\sqrt{n} \right) = 1-\alpha$$

$$\left(\bar{X}-Ls/\sqrt{n} \,,\; \bar{X}-Us/\sqrt{n} \right)$$

$$-L = U$$



$$\left(\bar{X} - t_{n-1,\alpha/2}\, S/\sqrt{n},\ \bar{X} + t_{n-1,1-\alpha/2}\right)$$

$(1-\alpha)100\%$ CI for μ

$$\bar{X} \pm t_{n-1,\alpha/2}\, S/\sqrt{n}$$

We are $(1-\alpha)100\%$ confident that the ~~true~~ population mean of ______ (mass volume) lies somewhere between $\left(\bar{X} - t_{n-1,\alpha/2} S/\sqrt{n},\ \bar{X} + t_{n-1,\alpha/2} S/\sqrt{n}\right)$